BC Calculus - 3-6 Chain Rule

Pg 153 # 1-33 odd, 41, 43, 53, 55, 56

(1) $y = \sin(3x+1)$

$y' = 3\cos(3x+1)$

(3) $y = \cos(\sqrt{3}\,x)$

$y' = -\sqrt{3}\cdot\sin(\sqrt{3}\cdot x)$

(5) $y = \left(\frac{\sin x}{1+\cos x}\right)^2$

$y' = 2\left(\frac{\sin x}{1+\cos x}\right)\cdot\left(\frac{\cos x(1+\cos x)+\sin^2 x}{(1+\cos x)^2}\right)$

$= \frac{2\sin x}{1+\cos x}\cdot\frac{1+\cos x}{(1+\cos x)^2}$

$= \frac{2\sin x}{(1+\cos x)^2}$

(7) $y = \cos(\sin x)$

$y' = -\sin(\sin x)\cdot\cos x$

(9) $s(t) = \cos\left(\frac{\pi}{2}-3t\right)$

$v(t) = -\sin\left(\frac{\pi}{2}-3t\right)\cdot -3$

$= 3\sin\left(\frac{\pi}{2}-3t\right)$

(11) $s(t) = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$

$v(t) = \frac{4}{\pi}\cos 3t - \frac{4}{\pi}\sin 5t$

(13) $y = (x+\sqrt{x})^{-2}$

$y' = -2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

(15) $y = \sin^{-5}x - \cos^3 x$

$y' = -5\sin^{-6}x\cdot\cos x + 3\cos^2 x\cdot\sin x$

(17) $y = \sin^3 x\cdot\tan 4x$

$y' = 4\sin^3 x\cdot\sec^2 4x + 3\sin^2 x\cdot\cos x\cdot\tan 4x$

(19) $y = 3(2x+1)^{-1/2}$

$y' = -\frac{3}{2}(2x+1)^{-3/2}\cdot 2$

$= -3(2x+1)^{-3/2}$

(21) $y = \sin^2(3x-2)$

$y' = 2\sin(3x-2)\cdot\cos(3x-2)\cdot 3$

$= 6\sin(3x-2)\cos(3x-2)$

$= 3\sin(6x-4)$ (double angle formula)

(23) $y = (1 + \cos^2 7x)^3$

$y' = 3(1 + \cos^2 7x)^2 \cdot 2\cos 7x \cdot -\sin 7x \cdot 7$

$= -42(1 + \cos^2 7x)^2 \cdot \cos 7x \cdot \sin 7x$

(25) $r = \tan(2 - \theta)$

$\frac{dr}{d\theta} = -\sec^2(2 - \theta)$

(27) $r = (\theta \cdot \sin\theta)^{1/2}$

$\frac{dr}{d\theta} = \frac{1}{2}(\theta \cdot \sin\theta)^{-1/2} \cdot (\theta\cos\theta + \sin\theta)$

$= \frac{\theta\cos\theta + \sin\theta}{2\sqrt{\theta \cdot \sin\theta}}$

(29) $y = \tan x$

$y' = \sec^2 x$

$y'' = 2\sec x \cdot \sec x \cdot \tan x$

$= 2\sec^2 x \cdot \tan x$

(31) $y = \cot(3x-1)$

$y' = -3\csc^2(3x-1)$

$y'' = -6\csc(3x-1)\cdot(-\csc(3x-1)\cdot\cot(3x-1))\cdot 3$

$= 18\csc^2(3x-1)\cdot\cot(3x-1)$

(33) $f(g(x)) = (\sqrt{x})^5 + 1 = x^{5/2} + 1$

$\frac{d}{dx}(f(g(x)))\Big|_{x=1} = \frac{5}{2}(1)^{3/2} = 1$

(41) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-2\sin t}$

$\frac{dy}{dx}\Big|_{t=\frac{\pi}{4}} = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -1$

$x(\frac{\pi}{4}) = \sqrt{2}$

$y(\frac{\pi}{4}) = \sqrt{2}$

$y - \sqrt{2} = -(x - \sqrt{2})$

(43) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{2\sec t \cdot \sec t \cdot \tan t}$

$= \frac{1}{2\tan t}$

$\frac{dy}{dx}\Big|_{x=-\frac{\pi}{4}} = -\frac{1}{2}$

$x(-\frac{\pi}{4}) = 1$

$y(-\frac{\pi}{4}) = -1$

$y + 1 = -\frac{1}{2}(x-1)$

(53) $y = \sin(\frac{x}{2})$

$y' = \frac{1}{2}\cdot\cos(\frac{x}{2})$

largest possible value of $\cos(\frac{x}{2})$ is 1,

$\therefore$ largest possible y' is $\frac{1}{2}(1) = \frac{1}{2}$

(55) $y = 2\tan\left(\frac{\pi x}{4}\right)$, $x = 1$

$y(1) = 2$

$y' = 2\sec^2\left(\frac{\pi x}{4}\right)\cdot\frac{\pi}{4}$

$= \frac{\pi}{2}\cdot\sec^2\left(\frac{\pi x}{4}\right)$

$\to y'(1) = \frac{\pi}{2}\cdot\sec^2\left(\frac{\pi}{4}\right) = \pi$

tangent: $y - 2 = \pi(x-1)$

normal: $y - 2 = -\frac{1}{\pi}(x-1)$

(56) a) $\frac{d}{dx}(2\cdot f(x))\Big|_{x=2} = 2\cdot f'(2) = \frac{2}{3}$

b) $\frac{d}{dx}(f(x)+g(x))\Big|_{x=3} = f'(3) + g'(3) = 2\pi + 5$

c) $\frac{d}{dx}(f(x)\cdot g(x))\Big|_{x=3} = f(3)\cdot g'(3) + f'(3)\cdot g(3) = 3\cdot 5 + 2\pi\cdot -4$

$= 15 - 8\pi$

d) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)\Big|_{x=2} = \frac{g(2)\cdot f'(2) - f(2)\cdot g'(2)}{[g(2)]^2} = \frac{2\cdot\frac{1}{3} - 8\cdot(-3)}{2^2} = \frac{\frac{2}{3}+24}{4}$

$= \frac{74}{12} = \frac{37}{6}$

e) $\frac{d}{dx}(f(g(x)))\Big|_{x=2} = f'(g(2))\cdot g'(2) = f'(2)\cdot -3 = \frac{1}{3}\cdot -3 = -1$

f) $\frac{d}{dx}(\sqrt{f(x)})\Big|_{x=2} = \frac{1}{2}(f(2))^{-1/2}\cdot f'(2) = \frac{1}{2\sqrt{8}}\cdot\frac{1}{3} = \frac{1}{6\sqrt{8}} = \frac{1}{12\sqrt{2}}$

g) $\frac{d}{dx}\left(\frac{1}{g^2(x)}\right)\Big|_{x=3} = -2\cdot(g(3))^{-3}\cdot g'(3) = \frac{-2}{(-4)^3}\cdot 5 = \frac{-10}{-64} = \frac{5}{32}$

h) $\frac{d}{dx}\left(\sqrt{f^2(x)+g^2(x)}\right)\Big|_{x=2} = \frac{1}{2}\left[f^2(2)+g^2(2)\right]^{-1/2}\cdot\left[2f(2)\cdot f'(2) + 2g(2)\cdot g'(2)\right]$

$= \frac{1}{2}(64+4)^{-1/2}\cdot\left(2\cdot 8\cdot\frac{1}{3} + 2\cdot 2\cdot -3\right)$

$= \frac{1}{2\sqrt{68}}\cdot\left(\frac{16}{3} - \frac{36}{3}\right) = \frac{-20}{6\sqrt{68}} = \frac{-20}{12\sqrt{17}} = \frac{-5}{3\sqrt{17}}$